THE MOST DIFFICULT LOGIC PUZZLES IN THE UNIVERSE

NO TRICKS, NO HINTS

ROI S. AHARON

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"LOGIC IS THE BEGINNING OF WISDOM, NOT THE END"

MR. SPOCK

INTRODUCTION

For as long as I can remember, I've enjoyed solving logic puzzles. They always felt like a rewarding, challenging workout for my brain – except for when I couldn't solve one, then it felt more like a sneaky little worm, setting up camp in the backyard of my mind!

During my many years of studying and teaching Mathematics in high-school and academia, I've encountered numerous types of difficult puzzles. This is a collection of the best ones.

Some of the puzzles are extremely challenging, and are even presented in interviews for highly classified tech positions. I didn't go to the interviews looking for a job, though – I preferred traveling the world, or teaching in a school, or playing my guitar or looking at the sky.
I only went for the puzzles.

The puzzles are ranked according to the following key:

∞ Difficult Puzzle
∞∞ Very Difficult Puzzle
∞∞∞ Extremely Difficult Puzzle
∞∞∞+ (Almost) Unsolvable Puzzle

All of the puzzles have a logical - and clever - solution, with no word games or tricks of any kind. Notice that ∞∞∞+ puzzles may incorporate some advanced mathematical principles from various fields, such as combinatorics, optimization and number theory.

The solutions are presented at the end of the book, so you won't be tempted to take a peek!

This book is for the high-school student who wants to improve their thinking skills; for the advanced mathematics student who wishes to practice what they learn; for the parent who wants to spend some offline quality time with their curious children; for the elderly who wish to keep their mind sharp.

Only the best puzzles in the universe are to be found in this book - a total of 42 puzzles. I'll take quality over quantity any day - and those 42 puzzles are sufficient for long, beautiful, challenging hours of thinking.

I hope you'll enjoy your way to the solution, and more importantly - find it!

Because it's not about the path. It's the outcome that matters.

Just kidding.

TIPS FOR SOLVING THE PUZZLES

First and foremost - take your time before reading the solution. Patience is still a virtue, isn't it?

In our modern day and age, we tend to look for quick solutions, and give up easily when we can't find them. However, solving one puzzle in this book can take up to a few days!

Try approaching the problem from different angles, let your mind rest for a while, and get back to the puzzle later.

Remember the phrase "sleep on it"? The mind sometimes does its processing while we do other things. It works - we really do have a beautiful mind.

You can share a puzzle with friends and family, think together, or ask someone (who's not as eager as you) to read the solution and give you a hint. Sharing is also a scientifically proven technique for improving the mind's capacity.

And most importantly - never give up.
No matter what's going on
Never give up.

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1 GENUINE COIN #1 ∞

I give you nine golden coins (yep, just like that)! Eight of them are counterfeits, and only one is genuine. The genuine coin weighs a tiny bit more than the counterfeits. You'll get the genuine coin all for yourself - only if you succeed to find out which one it is. You may use exactly two weighings on a balance scale. How will you find the genuine coin?

Once you find it, you can buy an organic meal for the nice elderly couple who live down the street.

2 THE EMPLOYEE ∞

You hired an ambitious teenager to clean your house. Your house contains seven rooms, and the teenager cleans one room per day. You and the teenager agree that the fee for their trouble is one gold bar. However, you and the teenager don't trust each other, so you must pay the teenager their daily proportional fee, at the end of each work day. (Just to be clear, the teenager should receive the proper fee at the end of the first day of work, at the end of the second day of work, and so on…). Had you been able to break the gold bar into seven pieces, you could have simply given him one piece every day. But how can you pay the teenager properly, without breaking the gold bar into more than three pieces?

After the job is done, you and the teenager need to have a little talk about trust…

3 WHO CALLS FIFTY? ∞

You and I are playing a game, in which each one of us calls out an integer, in turns. The first person who calls "50" wins. The rules of the game are as follows: the first person to call out a number, must call a number between 1 and 10. The next number to be called must be larger than the last one by 10 numbers at most (for example, if I called 7, you can call any number between 8 and 17). Same goes for the next turns - each number called can be larger than the previous one by 10 numbers at most. You are the first player. Devise a strategy in which the victory is yours, regardless of what I do.

The loser must give the winner 50 hugs.

4 SECRET SALARIES ∞

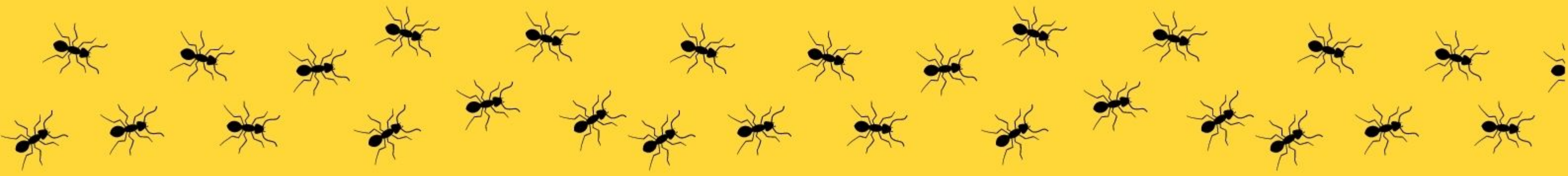
You and two of your friends have just received your weekly paychecks. The three of you wish to calculate the average amount of your salaries, but none of you is willing to tell any of the other two the exact amount of their salaries. Suggest a strategy in which the three of you are able to calculate the average salary — without any one having to tell anyone else their exact salary.

… It's a bit silly to keep your salary a secret - or to think that one can be measured according to how much money they make. It makes for an interesting puzzle though!

5 FAST ANTS ∞∞

1000 identical ants are randomly spread on a one-meter long plank. They can move towards one end of the plank or the other. The ants move at a constant speed of one-meter per minute. When two ants meet, they shake hands and reverse their direction. Assume that the shaking of hands is instantaneous and involves no time delay. If an ant reaches the end of the plank, it falls off to the ground. How long will it take until there are no ants left on the plank?

*No ants were hurt during the writing of this puzzle.



6 SECURED PACKAGE ∞

Plato has to send a top-secret package to Aristotle, so secret, that I don't even know what's in it. The package can be locked with several standard key-locks. Plato and Aristotle both have several locks and keys, but none of Aristotle's keys opens any of Plato's locks. Assume that if the package is sent unlocked, it is stolen. How can the package get to Aristotle safely, with him being able to open it?

7 THE DARK CAVE ∞

Dad, mom, son and daughter need to get from one side of a dark cave to the other. Dad takes 5 minutes to walk from one side to the other, mom takes 10 minutes, son takes 20 minutes and daughter takes 25 minutes. They mustn't walk in the dark. Yet, they only have one flashlight that lasts for 60 minutes. The cave is very narrow, so only 1 or 2 people can walk together at the same time. When two people cross the cave, sharing the flashlight, they travel at the slower person's speed. How can the family cross the cave safely?

It is only in this specific family that the men are faster than the women, and does not represent all families.

8 RACE HORSES ∞∞

A stable contains 25 race horses. You wish to determine which 3 horses are the fastest, and the order of their speed: who's 1st, who's 2nd and who's 3rd. Problem is, only 5 horses can race each other at a given time. How can you grade the 3 fastest horses within 7 races tops?

Wild, wild horses
Couldn't drag me away
Wild, wild horses
We'll ride them someday

9 TRIBE OF PROPHETS ∞∞∞

Once upon a time there was a little village. The village was populated by 30 married couples and a Chief. One day, the Chief gathered the village men and announced, "It has come to my attention that a prophecy has been given to certain women in the village. I have written the names of these women on notes. I will now hand each man one note, in which all the names of the women who have received the prophecy are listed, with one exception – his own wife will not be listed" – meaning that even if his wife is indeed one of the woman who received the prophecy, her name will not be listed on her husband's note. After handing out the notes the chief spoke to all the men, "I order that if one of you knows for a fact that your wife received the prophecy, you must play the drums at exactly midnight that day". The men took the notes and went to their tepees. After 2 nights passed quietly, a few drum sounds were heard on the third night, throughout the village. How many women had received the prophecy and how did their husbands know?

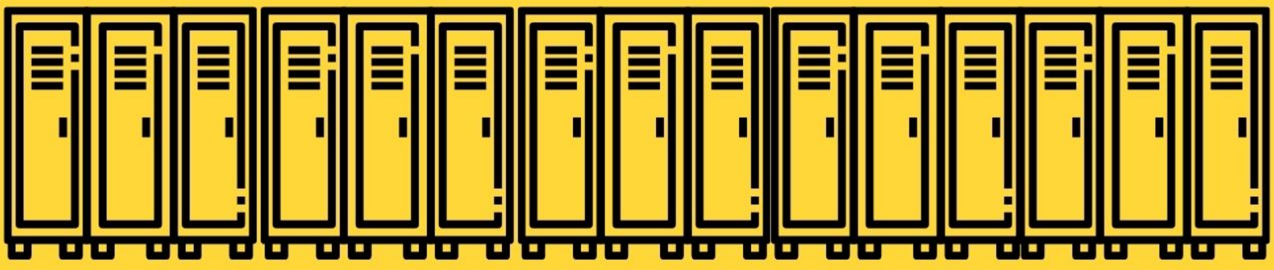
10 PRISONERS GETAWAY #1 ∞∞∞

The warder in *section A* of the infamous "Prison of the Mind," presented his 100 prisoners with a challenge. First, the warder will line up all prisoners into a column. This means that the rear prisoner sees the 99 prisoners in front of him; the next prisoner sees the 98 prisoners in front of him, and so on, up to the front prisoner who doesn't see any other prisoners in front of him. Then, the warder will put a hat on each prisoner's head - either black or white. None of the prisoners knows which color their hat is. The warder then passes next to each prisoner, beginning from the rear end and working his way to the front. As he passes, each prisoner, in their turn, has to shout "black!" or "white!" according to the color they believe their hat to be. If a particular prisoner is right, the warder sets that prisoner free. During the challenge, the prisoners cannot speak or move, but they can devise a strategy before the challenge begins. What strategy can lead to all prisoners being freed? Note that the rear prisoner gets pardoned (regardless of whether they got the answer right).

Kids - in real life, in order to be forgiven, you actually need to feel sorry. Being good at solving puzzles just ain’t gonna cut it.

11 SCHOOL LOCKERS ∞

The Music Academy of Utopia has 1000 students. Each student has a locker. The lockers are always closed at the beginning of the day and are numbered from 1 to 1000. Every day, the students come to the academy, one by one, according to the order of the lockers (the owner of locker no. 1 comes 1st, the owner of locker no. 2 comes 2nd and so on). Every student alters the state of their locker - if it's open, they close it, and if it's closed - they open it. The student also alters the state of every locker whose number is divisible by the number of their own locker. For example, the owner of locker no. 3 also alters the state of lockers no. 6, 9, 12, 15 and so on. Which of the lockers will remain open after the 1000th student finishes?



12 CUT THE DECK ∞∞

I take a standard deck of 52 cards, flip 10 random cards, put them back and shuffle the deck. I then hand the deck to a blind person. How can they split the deck into 2 decks, in such a way that each deck has the same number of upside down cards?

"It is only with the heart that one can see rightly; what is essential is invisible to the eye" - The Little Prince

13 ROPE-CLOCK ∞∞

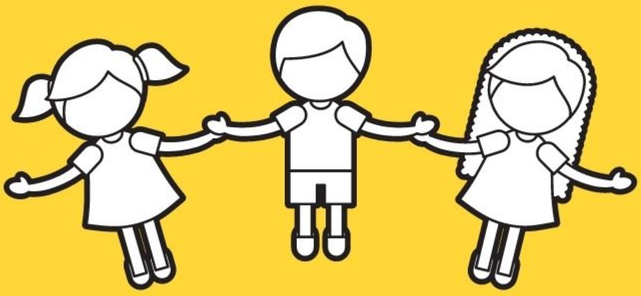
You have 2 ropes, each rope takes exactly 1 hour to burn when lit on one end. A rope doesn't necessarily burn at a permanent pace (for example, half of the rope doesn't necessarily burn for half an hour). How can you measure exactly 45 minutes, with only these 2 ropes and a lighter?

You have to be accurate, because your oven timer is broken, and you don't want your fair-trade chocolate-chip cookies burning in the oven!

14 CHILDREN'S AGES ∞∞

Two friends, Frida and Hilma, met on the street. Frida said, "the sum of my 3 children's ages is 13, and the product of their ages is the number written on the house across the street." Hilma answered, "Wait, I still can't figure out what their ages are." Frida said: "Oh, I forgot, the firstborn plays the piano." By then, Hilma knew the ages of all the children. What are the ages and how did Hilma find out?

Some people are just used to making things complicated.



15 WHICH CARD IS IT? ∞∞∞

Peter Pan has an ordinary deck of 52 cards. He randomly picks 5 cards and gives them to me. I look at the 5 cards, choose one card and keep it. I arrange the other 4 cards in whatever order I choose, and hand them to Tinker Bell. How can Tinker Bell know which card I'm holding? Notice that Tinker Bell and I are allowed to devise a strategy before the game begins.
It's called cooperation.

16 PRISONERS GETAWAY #2 ∞∞∞+

There are 500 prisoners in *section B* of the infamous "Prison of the Mind". The warder wants to give all of the prisoners a chance to be pardoned. The warder presents the prisoners with the following challenge, "I have a hat filled with notes, and all your names are written on the notes. I'll randomly choose a note from the hat and call the prisoner whose name is written on the note. All of you, by the way, must stay in your isolated cells, without seeing or hearing anything except for me calling you. When I call you, you must follow me into a room. In the room, there is a light-bulb connected to a switch. Notice that the light bulb is now turned-off. You may turn on, turn off or leave the light-bulb as it is. Then, you'll return to your cell. I'll return your name into the hat, and once again choose a note. The next prisoner entering the room will find the light-bulb in the same state as the last prisoner had left it, and they may turn on, turn off or leave the light-bulb as it is. I'll keep calling your names until one of you can tell me that they know for a fact that every single prisoner has already been inside the room at least once. You are also authorized to devise a strategy together before I begin". How will the prisoners know, at some point, that every single prisoner has already been inside the cell at least once? Disregard factors of duration and probability.

“Like a bird on a wire, like a drunk in a midnight choir, I have tried, in my way, to be free” - Leonard Cohen

17 GENUINE COIN #2 ∞

There are 10 bags of coins in front of you. Each bag has 10 coins in it. 9 bags contain genuine coins that weigh 10 grams each, and one bag contains counterfeit coins that weigh 9 grams each. You may open the bags and take as many coins as you choose. How can you find out which bag contains the counterfeit coins — with only one weighing, on a standard weighing scale?

After you solve the puzzle, you have another challenge - deciding how many coins you'll donate to charity, and how many will be used for… chocolate!

18 HANDSHAKES ∞∞

60 people attended a social event. Some of them shook hands with other guests. Prove that at least 2 people shook the same number of hands.
Then wash your hands.

19 PILLS FOR A BLIND MAN ∞

A blind man goes to the doctor. The doctor gives him a prescription to take 3 pills - one blue pill, one yellow pill and one white pill. Assume that the blind man is standing in front of a table. On the table there are 6 pills - two blue pills, two yellow pills and two white pills. How will the blind man be able to follow the doctor's prescription (one pill of each color)?

He was blind, but now he sees.

20 CUCUMBERS ∞∞

The farmers' market of Cucumbersville has 999 cucumbers & 10 empty bags. Mr. Cucumber wishes to place the cucumbers in the bags, so that when the next client comes in, they will be able to buy any number of cucumbers between 1-999 (without taking the cucumbers out of the bags). Help Mr. Cucumber decide how many cucumbers should go into each of the bags.

21 WHAT NUMBER COMES NEXT? #1 ∞∞

1,11, 21, 1211, 111221, 312211, ...

CONGRATULATIONS !

YOU MADE IT HALF WAY

22 HELP THE FARMER ∞

A farmer is on his way home with 3 companions: a wolf, a sheep and a head of lettuce (yes, lettuce can be a companion). On his way home he must cross a river. There is a boat on the farmer's side of the river, but the boat can only fit the farmer and one other companion at a time. Notice that if the wolf stays alone with the sheep, it eats it. If the sheep stays alone with the lettuce, it eats it. How can the farmer pass the river without losing any of his companions?

And more importantly, why does a farmer have a wolf...?

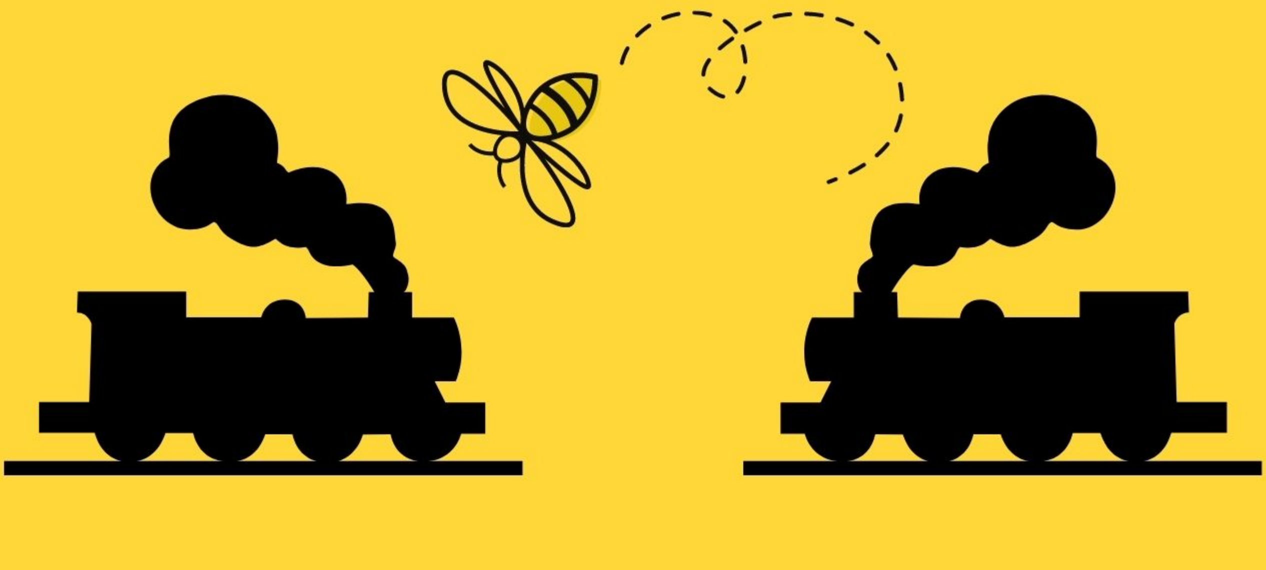
23 POISONED WINE ∞∞∞

The king of the island has 1000 wine barrels. It has come to his attention that one of the barrels is poisoned. The king has 10 loyal servants who are willing to taste the barrels and risk their lives for him. The problem is, the poison only takes effect a few hours after drinking it, and the wine must be used for a royal feast tonight at 8pm. Therefore, the servants only have enough time to drink once and wait for the results. Each servant may drink from one barrel or from a few barrels. The clever king had each servant drink according to a strategy he used, and after a few hours he found out which of the barrels was indeed the poisoned one. What was the king's strategy?

Aren't monarchies fantastic? Just makes things easier sometimes.

24 FAST BEE ∞

The distance between Neverland and Wonderland is 100 miles. They each have a station with a train. Two trains depart at the same time, one from Neverland and the other from Wonderland, and start moving towards each other at a speed of 100 mph each. A magical bee is standing on the train in Neverland. When the train departs, the bee flies at a speed of 300 mph towards the other train. When it gets to the 2nd train, it turns back and flies back to the 1st train, and goes back and forth until the trains meet. What is the total distance covered by the bee from the moment the trains depart until the moment they meet?



25 A PAIR OF SOCKS ∞∞

There are 3 boxes of socks in your drawer: a box of red socks, a box of blue socks, and a mixed box of both red and blue socks. The boxes are labeled "red", "blue", and "mixed," but none of the labels matches what's inside its drawer. How can you know which of the boxes contains which color of socks, if you are only allowed to take one sock from one box?

I personally prefer wearing un-matching socks.

26 HATS AND FRIENDS ∞∞

Three of my friends are standing in a column. Each friend only sees the person standing in front of them. I tell them I have 5 hats, 3 are green and 2 are brown. I turn off the lights and put a hat on each of their heads. I turn the lights back on, and ask them to tell me if they know what color their hat is. The rear friend doesn't know, the middle friend doesn't know, but the front friend knows what her hat color is. What is her hat color and how did she know?

My friends are so intelligent! And lucky for me, they have time for my weird hat games.

27 WHAT NUMBER COMES NEXT? #2 ∞

0, 1, 2, 720!, ...

!!!

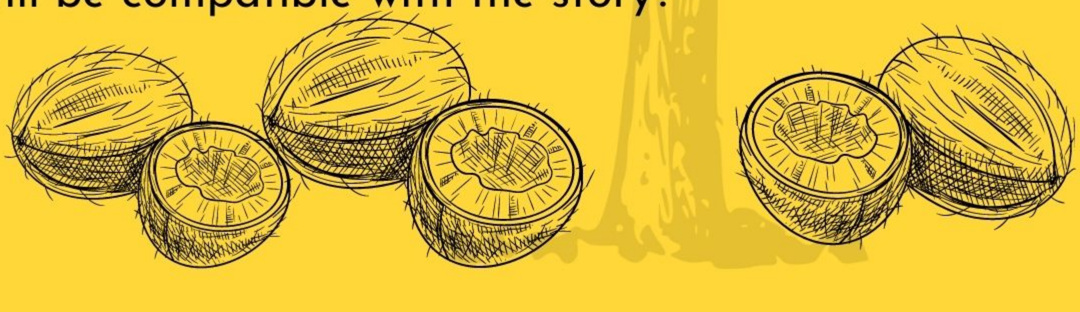
28 GREEDY PIRATES ∞∞

Five pirates have found a buried treasure on the coast of Somalia. The treasure contains 100 golden coins. They decide to split the treasure using the following method: The oldest pirate suggests how many coins each pirate gets. Then, the five pirates vote **for** or **against** his proposal. If 50% or more of them vote for the proposal, the coins are distributed in that way. If less than 50% vote for the proposal, then the oldest pirate loses his part of the treasure, as well as the right to vote. In such a case, the second oldest pirate will propose a new plan for the distribution, and the four remaining pirates will vote again, according to the same rules. As long as no plan was accepted, the process continues until the last pirate proposes a distribution plan all for himself. Assuming that all five pirates are greedy, rational, and good at logic puzzles, what would be the final distribution?

Also, what's a pirate's favorite letter? You'd think it would be arrr, but it's actually the C!

29 COCONUT ISLAND ∞∞

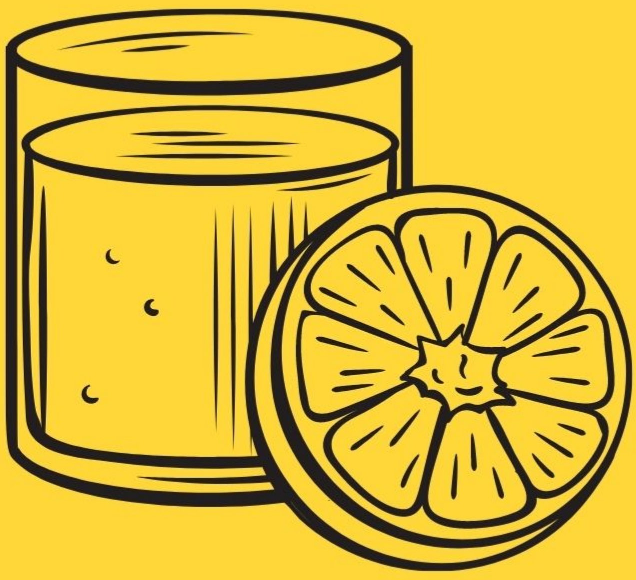
Ten people landed on a deserted island. Before sunset, they harvested all the coconuts they could find, and stacked them in a pile. They decided to go to sleep and divide the pile between them equally, the next morning.
At dawn, one person woke up, counted the coconuts, and discovered that there is only one coconut missing in order to divide the pile into 10 equal parts. Suddenly, she hears a noise coming from the trees, and sees a monkey holding one coconut. She tries to grab the coconut from the monkey, and chases it into the forest. This wakes up the next person, who sees there are only nine people left on the island. He tries to divide the pile into 9 equal parts, but quickly realizes that in order to do so, one coconut is missing. He notices a monkey holding one coconut. He tries to grab the coconut from the monkey, and chases it into the forest. The process proceeds - each person waking up, realizing that there is one coconut missing for an equal division of the pile into 8 parts, 7 parts, and so on. Then each person tries to grab a coconut from the monkey, and ends up chasing it into the forest. Once the last person wakes up, they have the whole pile for themselves. What is the smallest possible number of coconuts in the pile, that will be compatible with the story?



30 ORANGE JUICE ∞

For your upcoming birthday party, you decide to go to the local orchard and buy 4 gallons of fresh orange juice. You get to the little orchard shop and the farmer tells you she only has containers of 3 gallons and 5 gallons. How can you still measure exactly 4 gallons of orange juice?

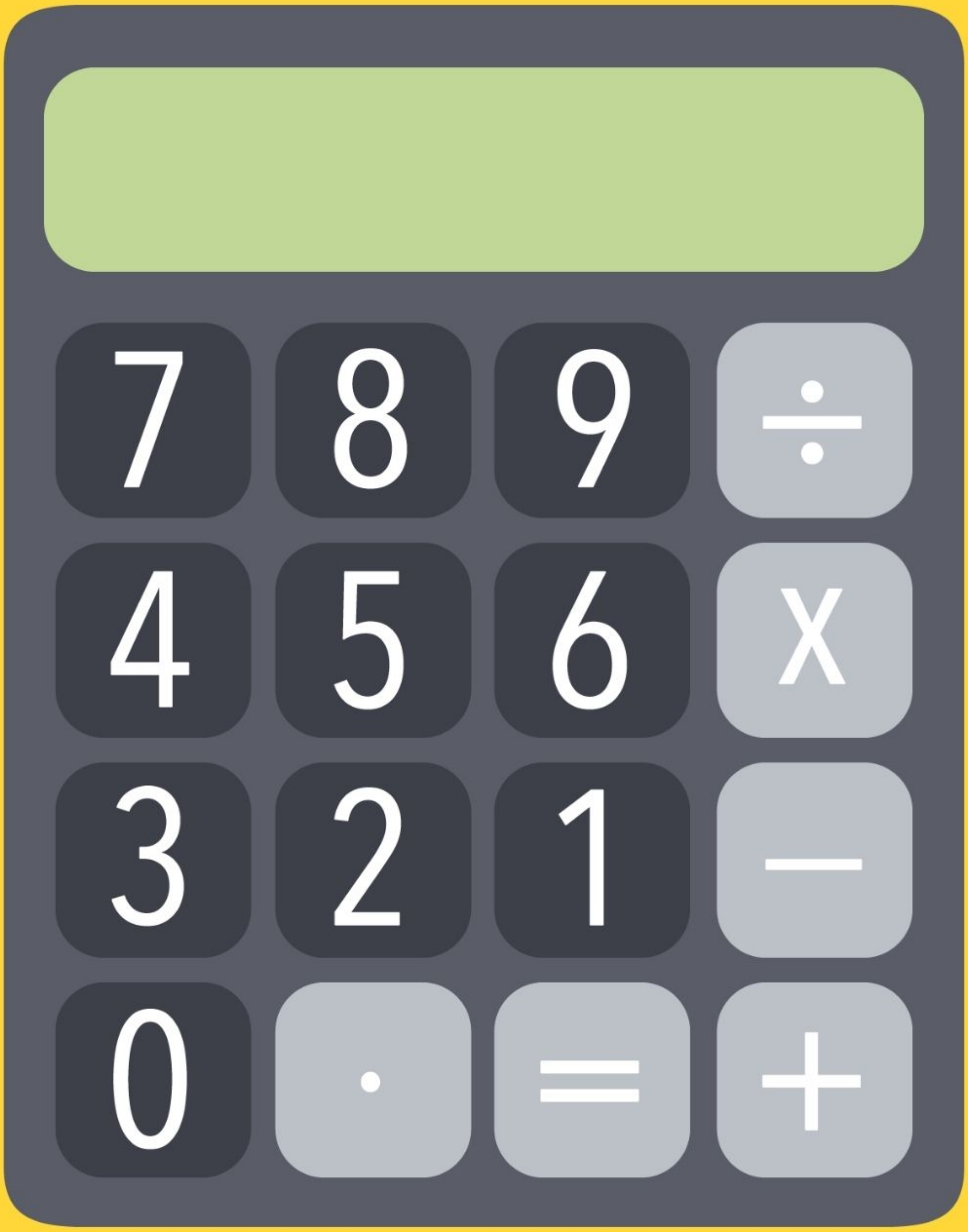
Happy orange birthday!



31 CAN'T MAKE 31 ∞

You wish to create a mathematical exercise, using only the numbers 1, 2, 3 & 4, and the four basic arithmetic operations: ×, ÷, +, - . You may use each number only once. Prove that the result of the exercise cannot be 31.

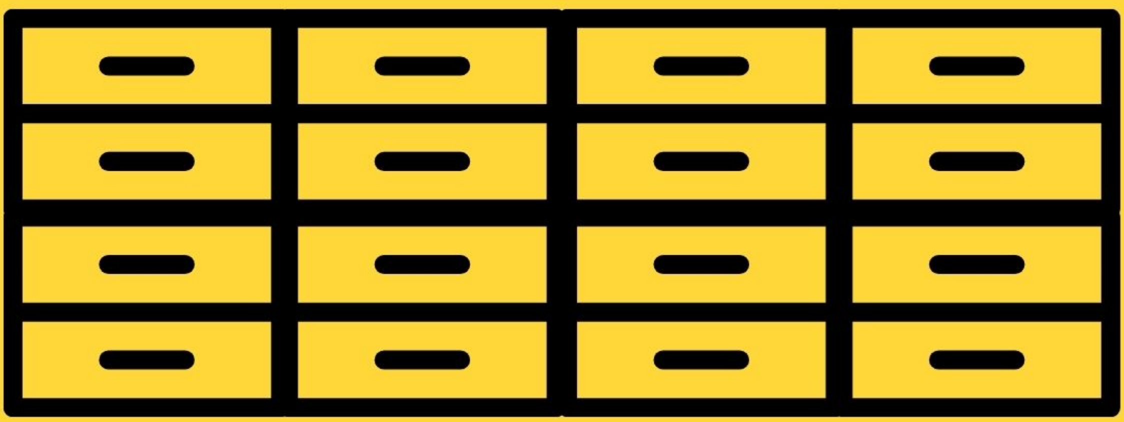
Other than that, you can do anything you set your mind to.



32 WHAT'S IN THE DRAWER? ∞∞∞

You and I receive a challenge from a common friend. Our friend prepared a room with 100 drawers, numbered from 1 to 100. Inside each drawer there is a note, and a number between 1 and 100 is written on it. The number on the note doesn't necessarily match the number on the drawer, and no two notes have the same number. Here is the challenge: One of us enters the room first. That person is allowed to look inside all of the drawers, and then, if they choose to do so, switch between two notes, meaning pull out 2 notes and then switch them. After that, our common friend chooses a number between 1 and 100. The second person, without communicating to the first one in any way, has to enter the room and find the note with that number. That person may only open 50 drawers at most. You and I are allowed to devise a strategy before the challenge begins. How can we ensure our success?

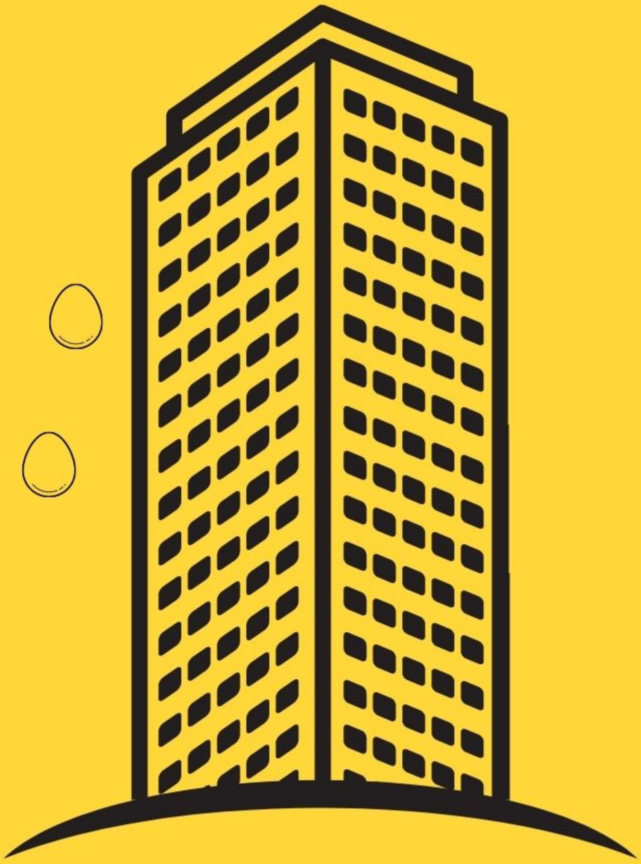
If we try our best, we've already succeeded.



33 HARD EGGS ∞∞∞

You are standing on a 100 story building, with two eggs in your basket. You wish to check how fragile the eggs are - meaning, which is the highest floor from which they can be dropped without breaking. You wish to use the least amount of drops as possible, especially since you already did your daily workout! What is the smallest number of drops that ensures you'll find the highest story from which the egg will not break? (both eggs have the same durability and will break starting from the same floor).

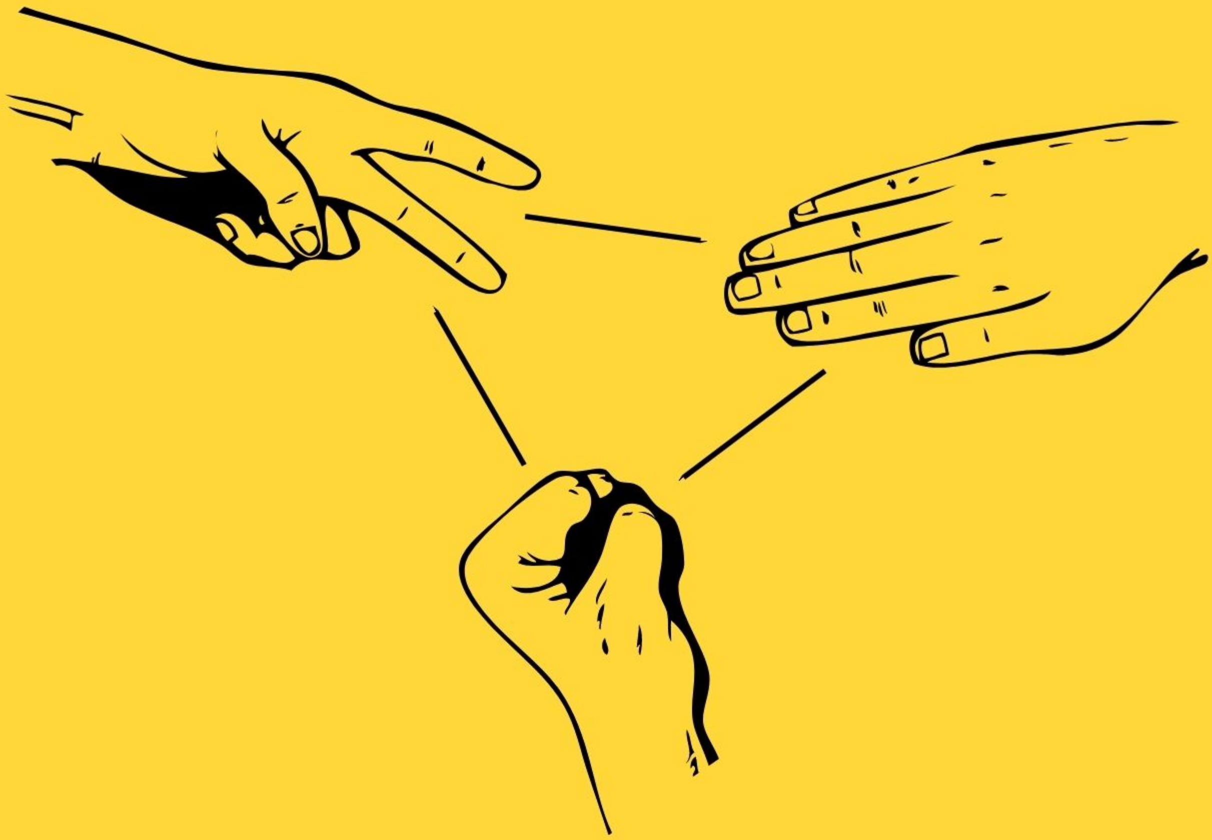
BTW, both eggs are free-range. Now find the solution so you can make me an omelette!



34 ROCK-PAPER-SCISSORS ∞∞

100 people are gathered together for the annual Rock-Paper-Scissors tournament. Each contestant plays all of the other contestants exactly once. Prove that at the end of the day, the players can be arranged in a line, so that each person has beaten the person on their left and lost to the person on their right.

Fun Fact - beginners, especially men, tend to choose Rock as their first move. RPS is more about psychology than chance.



35 A WISE MONK ∞

A wise monk had started to climb a mountain exactly at sunrise. On his way to the top, he stopped for food, drinks and rest. He got to the top of the mountain exactly at sunset. The next day he started to go down the mountain – on the exact same path as the one he took to climb up the day before. He started to walk exactly at sunrise, and a few hours later he got to the foothill of the mountain. The monk then pondered about his journey, and decided that there must have been a specific time during the day in which he was at the exact same location on the mountain – both while climbing up and while going down. How can he know this for sure?

One day I'll become a wise monk. Sounds like a cool job.

36 THE TRAPPED SNAIL ∞

A snail is chillin' inside a well whose depth is 20 feet. Every day, the snail climbs 3 feet up during the day, and slides 2 feet down during the night. After how many days will the snail be outside of the well? Assuming he is one of those persistent snails, of course.

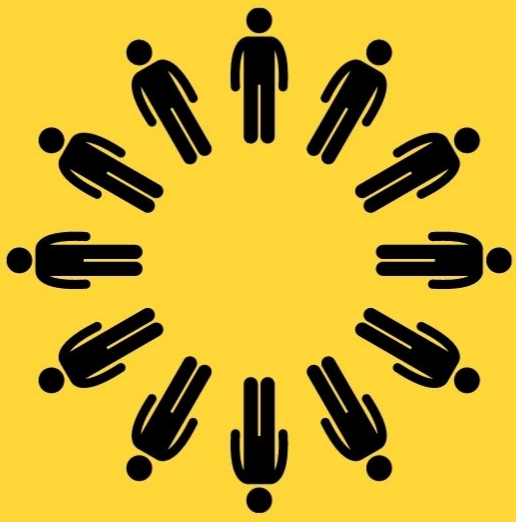
37 DUCK IN THE LAKE ∞∞∞+

Imagine a lake in the shape of a perfect circle. The circle's radius is 100 meters. There is a duck floating at the center of the lake, and a wolf standing on the bank of the lake. The duck's swimming speed is 1 meter per second, and the wolf's running speed is 4 meters per second. Furthermore, the wolf will always run towards the closest bit of shore to the duck (for example, if the duck swims towards the shore in the wolf's opposite direction, the wolf starts circling the lake towards the duck's destination). The wolf always takes the shortest route. If two paths take the wolf to the same place in equal times, it will choose one randomly. The duck's only wish is to get to the shore safely. What strategy would promise the duck a safe getaway to shore? Notice that the duck took some advanced math classes before swimming, so nonchalantly, in the wolf-surrounded lake.

38 PRISONERS GATEAWAY #3 ∞∞∞+

The warder, in *section* C of the infamous "Prison of the Mind," presents a puzzle to his ten prisoners. They must solve the puzzle in order to be set free. The 10 prisoners will enter 10 cells. The cells are built in a circle-shaped room, meaning every prisoner can see the other 9 cells. The warder will put a number between 1 and 10 on the top of each cell (a number may appear more than once). Then, every prisoner, who sees all of the numbers except for his own number, will write on a note the number he thinks his own cell has, and hand it over to the warder. If at least one prisoner will get his number right, they will all be set free. The prisoners can't communicate in any way during the puzzle but they may agree on a strategy before it starts. How can they promise that at least one prisoner writes the number he actually has?

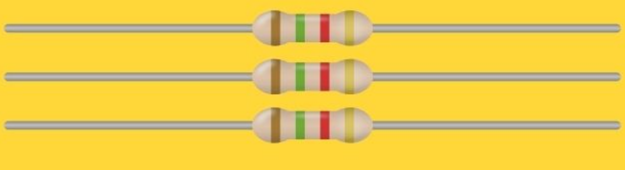
FREEDOM IS A STATE OF MIND



39 TRAINEE TECHNICIAN ∞∞∞

You are a trainee technician, and you are sent to correct an error made by your well-experienced employer. Your employer laid out a 10km long cable under the ground. The cable contains 120 wires. Despite her many years on the job, your employer forgot to label each individual wire - so you cannot tell which wire on one end matches which wire on the other end. You have a battery and a light-bulb to check conductivity, and as many labels and markers as you need. How can you match the two ends, of each and every one of the 120 wires, while covering as little distance as possible?

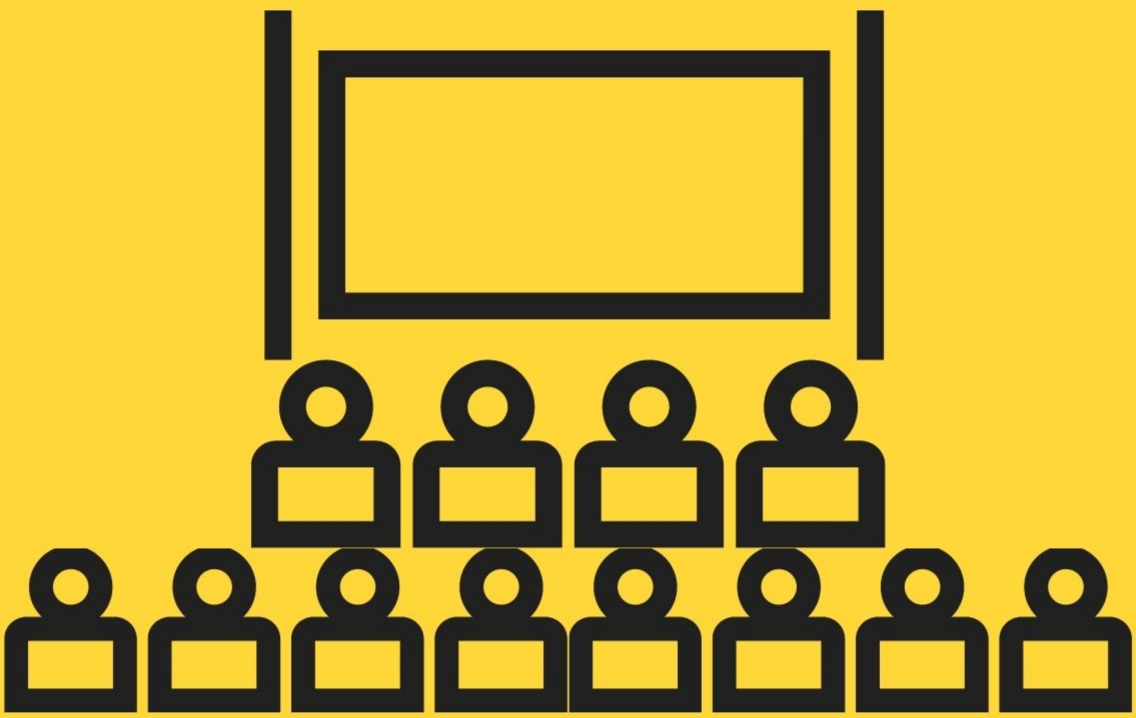
Whether we are trainees or well-experienced, there's always room for learning. That's the beauty of it.



40 MOVIE THEATRE ∞∞∞

There are 100 seats in the movie theatre, and 100 lady guests have purchased tickets with assigned seating. The guests enter the theatre one by one. The first guest is a rebellious one, and instead of sitting in her own seat, she randomly chooses a seat out of the 100 seats in the theatre. Each guest who enters after her follows this rule: in the instance that her seat is vacant, she will sit there. But, in the instance that her seat is not taken, she will randomly choose a seat from the remaining vacant ones. What is the probability that the last guest will sit in her preassigned seat?

Life would be boring if everyone just sits in their preassigned seats all the time.



41 GNOMES AND HATS ∞∞

Fifty gnomes are standing in the forest, at the end of a rainbow, outside of a cabin. A fairy comes and places a hat on each of the gnomes' heads. Some of the hats are blue and some are red. No gnome knows the color of his own hat, but is able to see all of the other hats. The gnomes must enter the cabin and arrange themselves in two groups - all of the blue hats on the one side and all of the red hats on the other. They cannot communicate in any way, but they are allowed to devise a strategy before the fairy places the hats on their heads. How will they do it?

Love is always the answer.

42 PRIME NUMBERS ∞∞∞+

You are presented with a random prime number "P". Prove that there exists a number, consisting of only the digits 1 and 0, that is divisible by that given prime number.

Fun Fact - prime numbers are used to secure almost every online purchase. So the next time you order a fair-trade T-shirt, thank 2, 3, 5, 7... although, it might take a while.

1001011

SOLUTIONS

1 GENUINE COIN #1 ∞

Divide the nine coins into three groups of three coins each. You now have three "trios." Choose two of the trios randomly, and put them on both sides of the scale. If one trio is heavier than the other, it means that the genuine coin is in that trio. If the scales balance, the genuine coin is in the third trio. Now you know which trio contains the genuine coin. Take the trio that contains the genuine coin, and choose two coins from that trio randomly. Place them on both sides of the scale. If one of the coins is heavier than the other, it is the genuine one. If both coins weigh the same, the third coin of the trio is the genuine one.

You got the real coin! It's all yours now. Use it wisely.

Break the gold bar into three pieces: the first piece is one-seventh of the bar, the second piece is two-sevenths of the bar, and the last one is four-sevenths of the bar . Than pay the teenager as follows:

1st day - give the teenager the one-seventh piece.
2nd day - take the one-seventh piece back and give the teenager the two-sevenths piece.
3rd day - give the teenager the one-seventh piece.
4th day - take the one-seventh and two-sevenths pieces back and give the teenager the four-sevenths piece.
5th day - give the teenager the one-seventh piece.
6th day - take the one-seventh piece back and give the teenager the two-sevenths piece.
7th day - give the teenager the one-seventh piece.

Personally, I like cleaning. Feels like I actually made a difference.

3 WHO CALLS FIFTY? ∞

Here is the order of number calling that assures your victory:

You: Call 6
Me: I can choose any number between 7-16
You: Whatever my choice was, you call 17
Me: 18-27
Y: 28
M: 29-38
Y: 39
M: 40-49
Y: 50!

Even in times of uncertainty, we always have a choice.

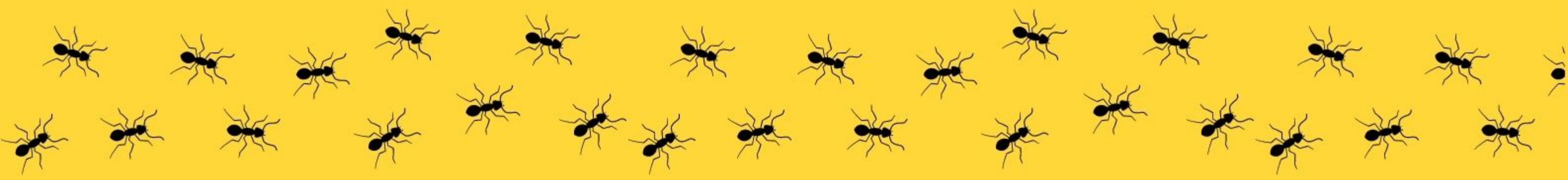
4 SECRET SALARIES ∞

You tell each of your friends only a certain part of your salary. For example, if your salary is 5000$, you whisper to one friend the number 2000$, and to the other one 3000$. Then, friend A tells friend B the sum of both their own salary and the number you told them combined. Friend B now adds up the number you told them, the number friend A told them, and their own number. Friend B now knows the sum of all 3 salaries without knowing the salary of each individual friend. They divide the number by three and tell everyone the average.

5 FAST ANTS ∞∞

Since all of the ants are identical, the situation in which two ants meet and reverse their direction can be replaced by the situation in which the two ants simply walk through each other (hypothetically speaking). In that hypothetical case, an ant wouldn't have had any obstacles in its path, and would have passed through the plank uninterrupted. Since the plank is one-meter long and the ants' speed is one meter per minute, the longest possible time that'll pass before each ant falls off the plank is one minute. Therefore, after one minute at most, there will be no ants on the plank.

Being the hardworking creatures they are, they immediately go looking for a new plank, just for fun.



6 SECURED PACKAGE ∞

Plato locks the package and sends it to Aristotle. Aristotle then locks the package with one of his own locks and sends it back to Plato. Plato then removes his own lock and sends the package back to Aristotle, which is now only locked by his own lock.

It turns out that inside the box there is a quote that Plato wrote: "Good actions give strength to ourselves and inspire good actions in others".

7 THE DARK CAVE ∞

Dad and Mom cross the bridge together - 10 minutes
Dad comes back alone - 5 minutes
Son and Daughter cross the bridge together - 25 minutes
Mom comes back alone - 10 minutes
Dad and the Mom cross the bridge together - 10 minutes

A total of 60 minutes.

By the way, it's mom who solved the puzzle.

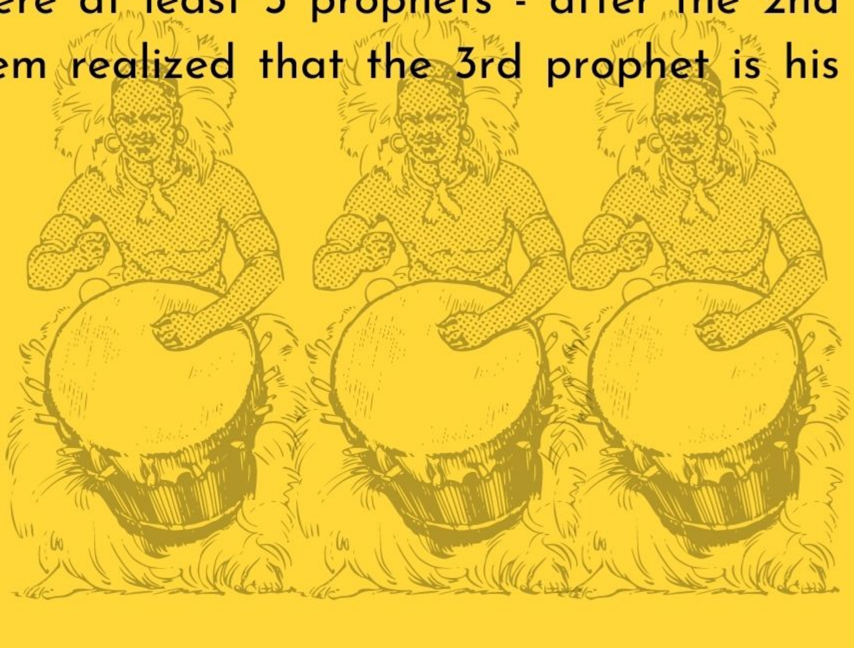
8 RACE HORSES ∞∞

First, divide the horses into 5 groups - A, B, C, D, and E - five horses per group. Perform a race for each group. Eliminate the 2 slowest horses in each group (as there are definitely at least 3 horses that are faster than them). Then perform a race between the 5 winners of the 1st round. Now, let's assume that for example, the top 3 horses of this round are the winners of groups A, B and C, in that order. Now you can eliminate all of the horses in groups D and E. The winner of group C is the 3rd fastest horse at best, so eliminate the two horses below him in group C. The winner of group B is the 2nd horse at best, so eliminate the 3rd place horse of group B. The winner of group A is the winner of the winners, so he is surely the fastest horse. You are now left with the 2nd and 3rd place horses of group A, 1st and 2nd place horses of group B, and the 1st place horse of group C, after 6 races were performed. Perform one more race – between those 5 horses – to determine which of the remaining horses are the 2nd and 3rd fastest ones.

After you're done, consider setting them free to roam in the wild.

9 TRIBE OF PROPHETS ∞∞∞

There are 3 woman who received the prophecy in the village, and here's how their husbands found out: every man knows that there is at least one prophet in the village. Had there been only one prophet, her husband would have received an empty note. He would have then concluded that his wife is the only prophet, and would have played his drums on the first night. The 1st night was quiet, so everyone in the village knew that there must be at least two prophets in the village. Had there been only two prophets, their husbands would have received notes with only one name on each note, concluded that their own wife is a prophet as well, and therefore played the drums on the 2nd night. The 2nd night was quiet, so everyone knew that there must have been at least 3 prophets in the village. There were some drum sounds on the 3rd night. Therefore, the number of prophets must be 3, because 3 men received a note with only 2 names written on it. Once they knew there were at least 3 prophets - after the 2nd night - each of them realized that the 3rd prophet is his own wife.

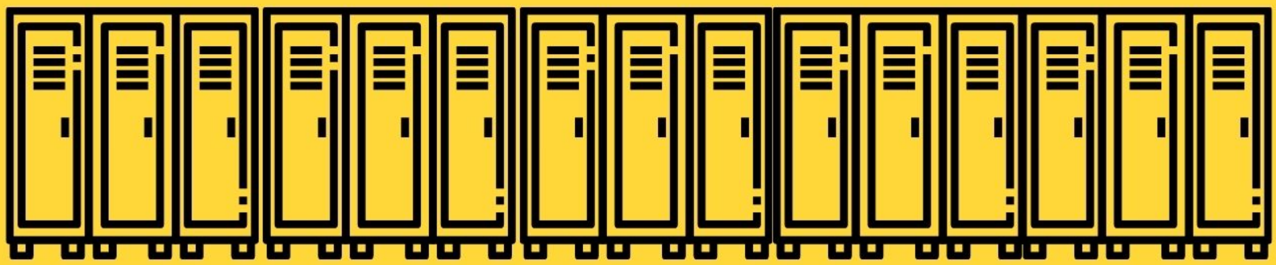


10 PRISONERS GETAWAY #1 ∞ ∞ ∞

The prisoners will follow the following strategy in order to be pardoned: the rear prisoner sees 99 hats, some are black and some are white. Since they are automatically pardoned, they can shout either "black" or "white", disregarding their own hat color. Notice that they must be seeing an odd number of hats of one of the colors (either black or white), and an even number of the other color. The prisoners agree that the rear prisoner shouts the name of the color that represents the **odd** number of hats.
For example: if the rear prisoner sees 47 black hats and 52 white hats, they shout: "black!". Then, if the next prisoner also sees an odd number of black hats, they know they must be wearing a white hat. If they see an even number of black hats, they know that they must be wearing a black hat.
The other prisoners also hear everything that's happening. For example, if the 2nd prisoner shouted "white!", the 3rd prisoner knows that the 2nd prisoner saw an odd number of black hats. The 3rd prisoner can then count the number of black hats they see, and deduce their own hat color accordingly. And so, by keeping track of each prisoner's shout, every prisoner will know for sure what kind of hat they have on.

11 SCHOOL LOCKERS ∞

The doors that will remain open, after all the students had gone passed the lockers are the ones whose number has an integer square root (1, 4, 9, 16...961). The doors that will be open at the end of the day, are the ones who have an odd number of divisors – and the only numbers that possess this quality are numbers with an integer square root.



12 CUT THE DECK ∞∞

The blind person should split the deck into 2 parts: one part containing 10 cards and the other part containing 42 cards. Now, if part A contains X upside-down cards, then part B must contain (10-X) upside-down cards (as there is a total of 10 upside down cards). The person should then simply flip the 10-card deck, which now also contains (10-X) upside-down cards.

13 ROPE-CLOCK ∞∞

Simultaneously, burn one rope on both ends and another rope on only one end. After exactly 30 minutes, the 1st rope will be completely burnt, as it was burned from both ends and has a total of 1 hour to burn. In addition, the 2nd rope will have 30 minutes more left to burn (as it was only burned on one end). At that moment, light the other end of the 2nd rope, and it will burn for exactly another 15 minutes.

"My candle burns at both ends;
It will not last the night;
But ah, my foes, and oh, my friends –
It gives a lovely light!"

— Edna St. Vincent Millay

14 CHILDREN'S AGES ∞∞

Hilma knows the children's ages after Frida tells her she has a firstborn child. Therefore, we can conclude that before that last piece of information, there was a trio that suited the information of the first sentence, with firstborn twins - either 1, 6 & 6 or 3, 5 &5 (these are the only two options with a sum of 13). There isn't any other series of 3 numbers whose sum is 13 and whose product is 3*5*5=75, so that can't be the number written on the house (as there has to be an option with the same product, that has a highest number). The product must therefore be 1*6*6=36. The only other option for the children's ages – that results in a sum of 13, a product of 36, and has no firstborn twins – is 2, 2, and 9.

15 WHICH CARD IS IT? ∞∞∞

First, Tinker Bell and I agree that all of the 52 cards go by some order (for example A♠, 2♠,... K♠, A♣,...A♥,... A♦,... K♦). When I receive 5 cards from Peter Pan, at least 2 of them must be of the same suit. Furthermore, one of these 2 cards must be, at most, 6 cards "higher" than the other one, (for example, in the case of 4♦ and K♦, 4♦ is 4 cards "higher" the K♦). I keep the card that is "higher" than the other. Tinker Bell and I also agree that I arrange the 4 remaining cards so that the top card has the same shape as the card I kept (and is, at most, 6 cards "lower" than the card I have). The other 3 cards are rated according to the order we set in advance. That way, they can be numbered as 1, 2 and 3, according to their predetermined order, (for example, if the remaining 3 cards are 5♣, 10♠ and 7♥ - we both know that 10♠ is 1st in order, 5♣ is second and 7♥ is third). These 3 cards can be arranged in 6 different patterns (1-2-3,1-3-2,2-13, etc.). We also number each of the different patterns with a number between 1 and 6 - ahead of time. Tinker Bell can therefore determine how much "higher" my card is in relation to the top card, according to the pattern in which I arranged the 3 remaining cards.

16 PRISONERS GETAWAY #2 ∞∞∞+

The prisoners choose a Joker - one prisoner who is given a special role. Every prisoner must turn on the light exactly once. If the prisoner already turned on the light once, this prisoner will not turn it on the next time he visits the room. If a certain prisoner hasn't gotten the chance to turn the light on yet, he'll continue entering the room until he finds the light bulb turned off, and turn it on. The Joker never turns on the light - he only turns it off. Since each prisoner turns on the light only once, after the Joker will turn off the light 499 times, they will know for a fact that every prisoner has visited the room at least once.

17 GENUINE COIN #2 ∞

You must number the bags from 1 to 10. Take 1 coin from bag no. 1, 2 coins from bag 2 and so on. Put the 55 coins on the scale. If all of the 55 coins were genuine, they would have weighed 550g. Since one bag contains counterfeit coins, and a counterfeit coin weighs 1g less than a genuine one, the number of grams missing to get to 550 is exactly the number of counterfeit coins, and therefore the number of the bag that contains them.

We will prove that the statement is true using a mathematical tool called **Proof By Contradiction**. Every person can shake between 0 to 59 hands. Let's assume that no 2 people out of the 60 guests shook the same number of hands. If this is the case, let's see if it makes sense. This would mean that one person shook 0 hands, one person shook 1 hand, one person shook 2 hands and so on (as there are 60 people and 60 different options). In this instance, one person had to shake 59 hands. But that must mean that this particular person shook everyone's hands, which is in contradiction to the scenario of one person shaking 0 hands. Therefore, the original assumption of no 2 people shaking the same number of hands – doesn't make sense. In other words, there had to be at least 2 people who shook the same number of hands.
Q.E.D.

19 PILLS FOR A BLIND MAN ∞

The blind man should simply split each pill into 2 pieces, and take half of each pill, in order to get to a total of one blue pill, one yellow pill and one white pill.

20 CUCUMBERS ∞∞

The customer must be able to buy any possible number of cucumbers between 1-999. First of all, there must be one bag that contains only 1 cucumber, otherwise it would have been impossible to buy only 1 cucumber. The next bag has to contain 2 cucumbers, for similar reasons. The next bag, however, doesn't need to contain 3 cucumbers. A client who wishes to buy 3 cucumbers can simply grab the 1 cucumber bag and the 2 cucumbers bag. Therefore, the next bag must contain 4 cucumbers. With those three bags, the client is able to buy 1, 2, 3 (1+2), 4, 5 (1+4), 6 (2+4) or 7 (1+2+4) cucumbers. The next bag should therefore contain 8 cucumbers, thus covering all of the options between 1-15. We continue the process, using powers of 2; the bags will therefore contain 1, 2, 4, 8, 16, 32, 64, 128, 256 & 512 cucumbers, thus covering all of the numbers between 1-999.

With all this cucumber talk, I get a sudden craving for zucchini.

21 WHAT NUMBER COMES NEXT? #1 ∞∞

The next number is 13112221. The pattern is as follows: each number tells us how many times a digit appeared in the previous number, in order. The digits for the last number (312211) are, by order: one 3, one 1, two 2's, two 1's. Therefore, the next number is 13112221.

22 HELP THE FARMER ∞

The farmer crosses the river with the sheep and leaves the wolf and the lettuce together on the 1st side of the river bank. He then leaves the sheep on the 2nd side of the river bank, and comes back. Then he takes the wolf to the 2nd side of the bank, leaves him there but takes the sheep back with him. He leaves the sheep on the 1st bank of the river and takes the lettuce. He leaves the lettuce and the wolf on the 2nd side of the bank and comes back for the sheep.

One day, the wolf shall dwell with the lamb.

23 POISONED WINE ∞∞∞

The solution involves some principles of **Combinatorics**. Assume that the servants are numbered from 1 to 10. First of all, let's check how many different combinations of servants are possible, (for example: servants 3, 5 and 7 are considered to be one combination. Servants 3, 6, 8, 9 and 10 are another combination, etc.). Each servant can either be a part of a specific combination or not be a part of it, so there are 2 options for each servant regarding a specific combination. There are 10 servants. Therefore, there are 2^10=1024 possible combinations of servants. The king therefore has enough combinations in order to match each barrell with a unique combination. He writes the combination on the barrel and lets those servants, and only those servants, sip from the barrel. Before the royal feast, the servants who were poisoned will be the ones whose numbers are written on the poisoned barrel, and them only. There is only one barrel that matches that specific combination of servants, and that is the poisoned one.

24 FAST BEE ∞

The distance between Neverland and Wonderland is 100 miles, and each train is moving at a speed of 100 mph. Therefore, the trains will meet half an hour after they started to move towards each other (each train covering a distance of 50 miles). The bee flies from train to train, from the moment the trains begins to move until the moment they meet. So, the duration of the bee's flight is also half an hour. It flies at a speed of 300 mph, meaning it will cover a total distance of 150 miles.

25 A PAIR OF SOCKS ∞∞

You must take a sock from the box labeled "mixed." Assume that the sock you took is red. The label "mixed" is false; meaning the correct label for this box is "red". You move the label from the box labeled "red" to the one formerly labeled "mixed". The box that is labeled "blue" is also false, so you move the label "blue" to the unlabeled box (formerly labeled "red"). Then you simply label the last box "mixed". Had you taken a blue sock in the beginning, the steps would have been exactly the same.

26 HATS AND FRIENDS ∞∞

The front friend knew her hat was green, and here's how: The friend in the rear of the line sees her 2 friends in front of her. Since she didn't know the color of her hat, the 2 front friends have either: 2 green hats or one green hat and one brown hat. If both hats were brown, the rear friend would have known that she is wearing a green hat. Since the friend standing at the rear of the line didn't know what her hat color was, the 2nd friend from behind knows that her and the front friend are wearing either 2 green hats or one green hat and one brown hat. Had the 2nd friend seen a brown hat on the front friend, she would have known that her own hat color was green. Since she didn't know her own hat color, the front friend concluded that her own hat must be green.

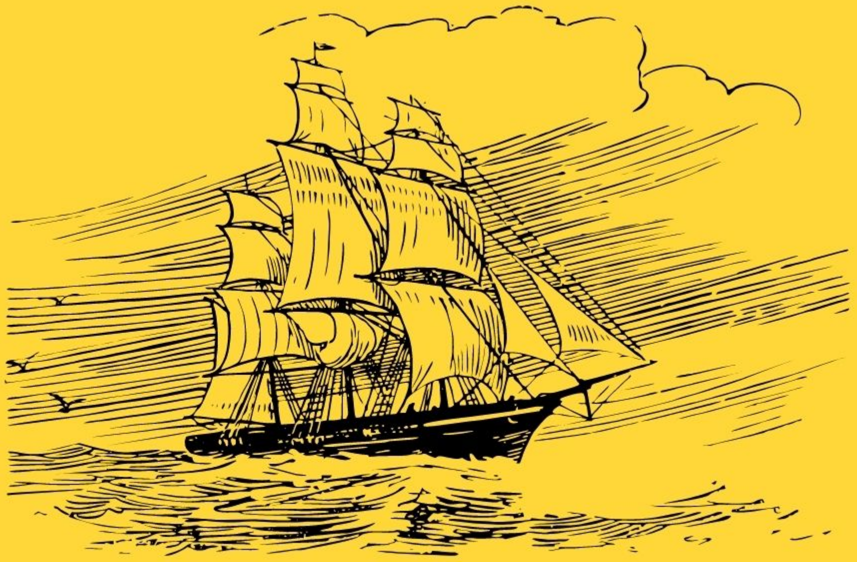
27 WHAT NUMBER COMES NEXT? #2 ∞

The series is: 0, 1!, 2!!, 3!!!, so the next number is 4!!!! (a huuuuge number!).

!!!

28 GREEDY PIRATES ∞∞

Let's look at the problem from end to beginning. If only the two last pirates remain, the fourth pirate will give himself all of the 100 coins, as his own vote will be sufficient for the proposal to be approved. Therefore, it's sufficient for the third pirate, on his turn, to offer 1 coin to the fifth pirate, in order to get his support and have the majority of votes. Then again - that would make it sufficient for the second pirate, on his turn, to offer 1 coin to the fourth pirate to get *his* support. Finally, that makes it sufficient for the first pirate to offer 1 coin to the third pirate and 1 coin to the fifth pirate — in order for it to be worthwhile for them to vote for his proposal. In conclusion, the distribution will be 98:0:1:0:1.



29 COCONUT ISLAND ∞∞

The solution incorporates principles of **Number Theory**. The number of coconuts in the pile, plus the monkey's coconut, add up to a number that has to be divisible by each and every number between 1-10. The number we are looking for is therefore one number less than the LCM (Least Common Multiple) of all the numbers from 1 to 10. In order to find it, we use Prime Factorization.

10=2*5

9=3*3

8=2*2*2

7=7

6=2*3

5=5

4=2*2

3=3

2=2

1=1

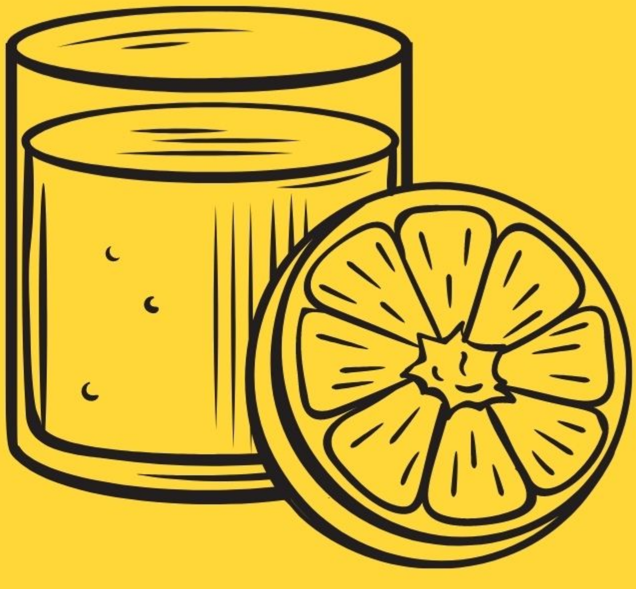
It's sufficient to multiply three 2s, two 3s, one 5 and one 7: 2*2*2*3*3*5*7=2520.

Therefore, there were 2519 coconuts in the pile.

Notice that if 1 coconut is added to the pile, it can be divided by each and every number between 1-10.

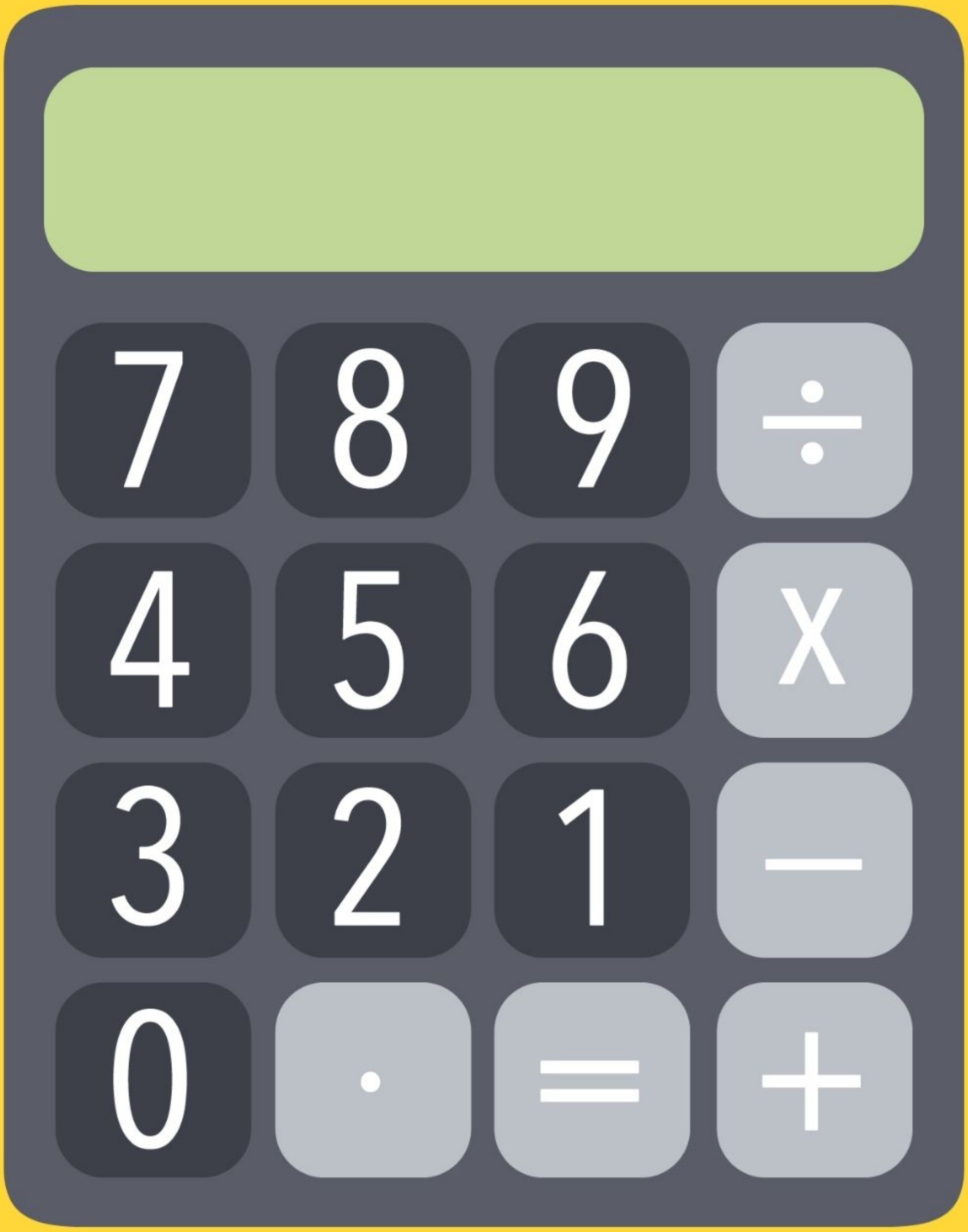
30 ORANGE JUICE ∞

Fill the 3 gallon container and pour its contents into the 5 gallon container. Fill the 3 gallon container again, and using it, fill the 5 gallon container up to the top. You used 2 more gallons, so the small container now has 1 gallon left inside of it. Get a new 5 gallon container and pour the 1 gallon container into it. Fill up the 3 gallon container one last time, and pour it into the 5 gallon one, for a total of 4 gallons.



31 CAN'T MAKE 31 ∞

The exercise that will generate the highest outcome is 1×2×3×4=24. Any other arithmetic operation between the numbers will only generate numbers lower than 24. Therefore, an exercise created according to the rules presented in this puzzle, cannot generate any numbers that are higher than 24 - including the number 31.

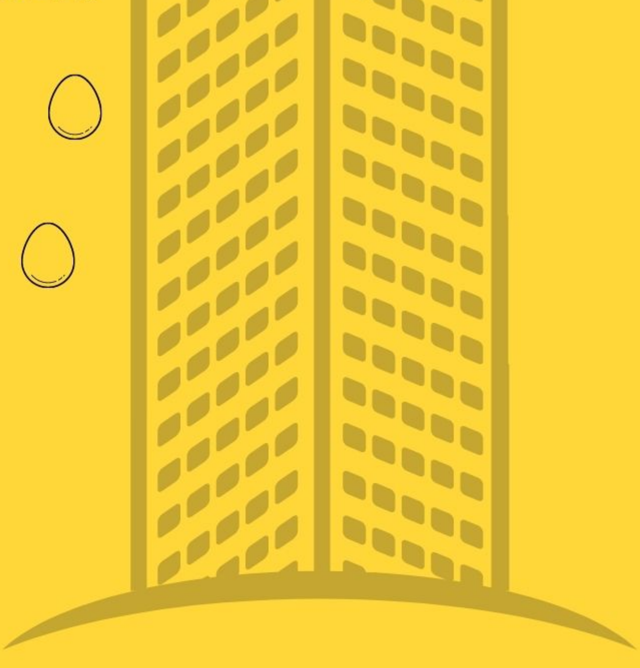


32 WHAT'S IN THE DRAWER? ∞∞∞

The solution involves principles of **Discrete Mathematics**. Let's say I go inside the room, and open drawer no. 4. I look at the note that is inside it - let's say it has the number 72 written on it. I then open drawer no. 72, and look at the note inside *that* drawer. Let's say it has the number 16 written on it. I then open drawer no. 16, look at the number that's written on note that's inside it, open the drawer that has the same number as the note, and so on. At some point I will find the note with the number "4" on it, and will go back to drawer no. 4 (It can happen after 100 drawers, after 8 drawers and even after one drawer). Let's call this a "chain" of drawers: starting from a specific drawer, and following the notes until getting back to that very same drawer. The length of a chain can therefore be anything between 1 and 100. Notice: in case there is a chain that is longer than 50 drawers, it's the only one (as there are only 100 drawers in total). Since I'm allowed to look at all the drawers, I can see all the different chains. If I see a chain that is larger than 50, I simply switch one of the notes in the middle of the chain with the last one, to create 2 different chains, both shorter than 50. At that point, all of the chains are shorter than 50 drawers. You can then enter the room, open the drawer with the same number that's written on the note our friend handed to you, and start following the chain. After less than 50 drawers, you will find the note carrying the number you're looking for.

33 HARD EGGS ∞∞∞

The solution involves principles of **Optimization**. If you drop the egg from a certain floor and it breaks, you will have to go through all of the floors, starting from the one you know for sure won't break the egg, until the one where it first breaks. The optimal way to do that is to start on the 14th floor. If the egg breaks, you will have to try a maximum of 13 drops with the other egg, in order to know which is the highest possible floor that won't break the egg. All together - 14 drops at most. If it doesn't break at the 14th floor, drop the egg from the 27th floor (13 floors higher). In case it breaks, you will have to check all the floors from the 15th floor to the 26th. That's 12 floors, plus 2 drops already used - a total of 14 drops. In case it doesn't break, drop the egg from the 39th floor (12 floors higher). continue this process with the 50th floor, 60th, 69th, 77th, 84th, 90th, 95th, and 99th floor. The maximum number of drops will always be 14.



34 ROCK-PAPER-SCISSORS ∞∞

The solution involves the principle of **Induction**. In case we have only 2 people playing, it would have been easy to arrange them so that the winner is standing to the right of the loser. Now, let's assume that we can arrange X people in that way, and prove that we can also arrange X+1 people, accordingly. We will arrange X people out of the X+1 (we assumed that we are able to do that). Now, we only have to prove we can fit the last person in the line properly. Let's call that person Maya. In case Maya lost to the person standing on the far left, she can just stand to their left, and we're done. Otherwise, she can stand to their right. If Maya lost to the 2nd person on the line, she can stay in her place, and we're done. Otherwise she can move one more step to the right. Maya continues the process until there is someone to her right that beat her. If no one beat her, she will end up standing to the right of the last person in line. So we have proved that the arrangement is possible for 2 people, and that if it's possible for X people, it's necessarily possible for X+1 people. Therefore, since it's possible to arrange 2 people, it's also possible to arrange 3 people. Since it's possible to arrange 3 people, it's possible to arrange 4 people, and so on. According to the law of induction, it's possible to arrange – in that way – any number of people playing.

35 A WISE MONK ∞

The monk realized that if two men, one on the top of the mountain and one at the bottom, had both started walking on the same day at sunrise, each of them following the same path, they would have certainly met. Therefore, the moment in which the monk would theoretically "meet" himself, is the moment in which he was in the same place – both while climbing up and while going down.

The answer was, all along, the moment the monk meets himself. Now that's what I call Zen.

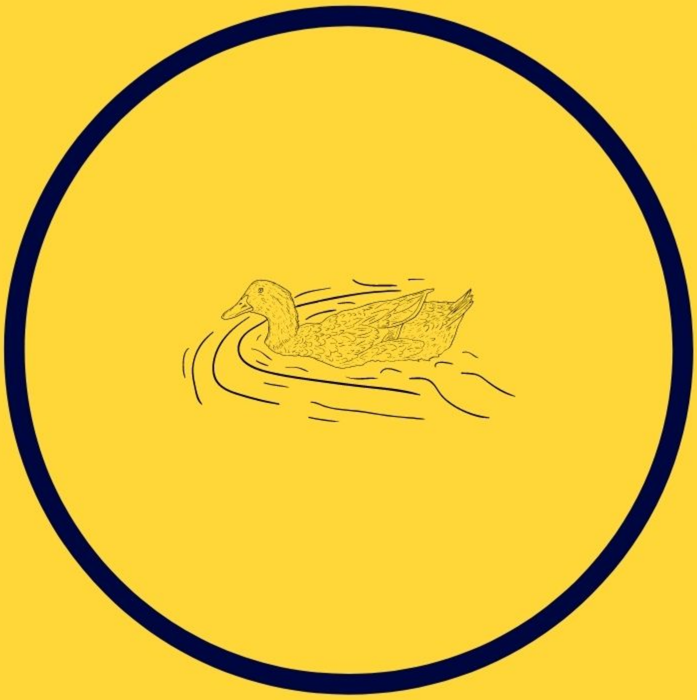
36 THE TRAPPED SNAIL ∞

The snail is seemingly climbing a total of 1 foot every day, meaning it will get out of the well within 20 days. But notice that after 17 days, the snail's location is 17 feet up the well. On the 18th day it will climb the extra 3 feet and get out, before dropping the 2 feet he usually does. In conclusion, the snail will take 18 days to get out of the well.

I am snailing, I am snailing...

37 DUCK IN THE LAKE ∞∞∞+

The duck must swim on the perimeter of a circle, that has the same center as the lake, and a radius of a tiny bit less than 25 meters. The perimeter of this circle is a tiny bit less than 50π (25*2π), while the perimeter of the lake is 100*2π=200π. Therefore, the duck is faster on his circle in relation to the wolf (since the lake's perimeter is more than 4 times the circle's perimeter). The duck can therefore swim in circles, while the wolf is chasing him on the bank, until they are located at radially opposite ends. Once the wolf is on the radially opposite end from the duck, the duck can take the shortest route to shore. Since the duck only has a tiny bit more than 75 meters to shore (a radius of 100 minus a radius of tiny bit less than 25), it will reach there before the wolf, who has to run 100π meters to where the duck gets to the shore (more than 4 times the duck's way to shore).



38 PRISONERS GATEAWAY #3 ∞∞∞+

The solution uses both the **Pigeonhole Principle** and **Modular Arithmetics**. Every prisoner can calculate the sum of all of the other prisoners' numbers. He does not know the total sum, which includes his own number. Nevertheless, the total sum exists, and has the following two characteristics:

1. The total sum's unit digit is between 0-9.
2. The total sum is between 1 and 10 numbers higher than the sum each prisoner sees (since his own number, is between 1 and 10 and is missing to complete the total sum).

Before the challenge begins, the prisoners decide, in advance, that each one of them will receive another number between 1 and 10 – regardless of the number that will be given to them by the warder.

Each prisoner receives a different number. After each prisoner sees the sum of his companions' numbers, he writes down the number that will complete this sum, to a total sum with the unit digit he received in advance. For example: if a prisoner sees a sum of 54, and his number (that was given in advance) is 6, he will write down the number 2, that completes a total sum of 56. That way, every prisoner will write down a number that completes a different total sum - and together they will cover all 10 different possibilities for the total sum. One of the sums must be correct - due to characteristic 2. The prisoner that received a number in advance, that matched the unit digit of the total sum, would therefore write down his own correct number.

39 TRAINEE TECHNICIAN ∞∞∞

You can label all of the wires, covering only 20km of distance! This is how: start on one end of the cable. Plug the battery into one wire randomly. Then pair up 118 wires randomly into 59 separate pairs, and leave one wire as is. Now go to the other end of the cable. Using the light bulb, check which wire is connected to the battery, and label it as no. "1".
Randomly choose another wire, label it as no. "2", and connect it to wire no. "1". The battery is now also connected to wire no. "2". Furthermore, wire no. "2" is connected to another wire on the other end. You can therefore check which one, using the light bulb. Once you find it, label it as no. "3". Then, choose another wire randomly, label it as number "4", and connect it to wire no. "3". In the same way as before, search for its pair using the light bulb, label the pair as number "5" and continue the process until you cover all the wires. Notice that, at some point, you will choose a wire that isn't connected to any of the others - that is the wire you left as is. You label it as number "120" and continue the process with a different wire. Once all of the wires are labeled on that end, and are also connected (wire no. "1" connected to no. "2", no. "3" connected to no. "4" and so on), go back to the other end. Make sure you know which wire is paired with which, and then disconnect all of them. Only wire no. "2" is now connected to the battery thru wire no. "1", and you can find it using the light bulb. You then connect it again to its original pair, which is wire no. "3". Wire no. 3 is connected on the other end to wire no. "4", so you can find it, and reconnect it to its pair, which is wire no. "5". You continue this process until all of the wires are labeled.

40 MOVIE THEATRE ∞∞∞

The solution uses a **Tree Diagram (probability)**. The key is to calculate the probability of the last guest to end up sitting in their predesignated sit - for each of the 100 possible choices the first guest has.

To begin with, each choice has a probability of 1/100.

Firstly, in case the first guest randomly chooses his own seat at the beginning, the last person will certainly find his own seat. That occurrence has a 1/100 chance of happening.

Second, in case the first guest chooses the last guest's seat, the chance of the last guest sitting in their seat is 0.

Now let's look at all of the remaining 98 options. In case the first guest chooses the 99th person's seat, all the guests – up until the 99th guest – will sit in their own seats. The 99th person will randomly choose between the first guest's seat and the 100th person's seat (both remained empty so far). The probability of the 100th person to sit in his seat is then ½. This must be multiplied by 1/100, which is the initial probability of the 1st guest choosing the 99th person's sit. We then receive a total of 1/200 chance of that occurrence.

In case the first guest chooses the 98th person's seat, the 98th person will randomly choose between 3 seats; if he chooses the 1st guest's seat, the last person will sit in their seat. If he chooses the 99th person's seat, the last person will have a chance of ½ to take his own seat. If he chooses the last person's seat, the last person will have 0 chances of getting his seat. The probability in that scenario will be ⅓*1+⅓*½+⅓*0= ½ . This should be multiplied by 1/100, to receive a total of 1/200 chances of that occurrence. Similarly, the probability will always be 1/200, for each of the 98 seats (that aren't the last person's seat nor the first person's seat).

Now, let's add up all of the probabilities: 1/100+0+98*1/200= ½ . Therefore, the probability for the last person to sit in their predesignated seat is exactly ½.

41 GNOMES AND HATS ∞∞

The gnomes agree to go inside the cabin one by one. The 1st gnome stands in the middle of the room. The 2nd gnome stands to his right. The third gnome comes inside. If he sees two hats of the same color, he stands to their right; if he sees two different colors, he stands in between them, so that whatever color his hat may be, he will stand next to the gnome who has the same color hat as him. This process continues, as each gnome goes inside the cabin, he will stand in between the two gnomes who have different colored hats. Once all of the gnomes are inside, the blue hatted gnomes will be standing on one side of the line, and the red hatted gnomes on the other.

The solution uses the **Pigeonhole Principle** and **Modular Arithmetics**. Let's look at the series of powers of ten: 1, 10, 100, 1000, and so on. Whatever our given prime number may be, we may divide each of the numbers in the series by it. For each division, we will get a remainder that is between 0 and P-1. This series of powers of 10 is infinite, so we can divide as many times as we want and receive an infinite number of remainders. According to the Pigeonhole Principle, there must be at least one remainder that will repeat itself an infinite amount of times (otherwise there would be a finite number of each remainder, in contradiction to the fact that there is an infinite number of remainders). We can therefore calculate the sum of P different numbers from the series, all having the same remainder. The sum is a number consisting only of 1s and 0s, and is definitely divisible by P, as it only consists of a portion that is divisible by P and P times the remainder we chose. Q.E.D.

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"EVERYTHING THAT HAS A BEGINNING, HAS AN END"

THE ORACLE

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ABOUT THE AUTHOR

Roi S. Aharon is a Mathematics professor, an educator, a writer and a social entrepreneur. Based in Tel-Aviv, Roi is working in various fields, in order to help individuals develop their minds and hearts, in the face of current global challenges.